IRJSE Int. Res. J. of Science & Engineering, Volume 13 (5) 2025
ISSN: 2322-0015
SJIF Impact Factor 6.137
https://doi.org/10.5281/zenodo.17166809
RESEARCH ARTICLE OPEN ACCESS

Foundations of Set-Theoretic Granular Computing in Modern Data Analysis

Rahul A. Parshuramkar and Arpit A. Kharwade*

Mahatma Gandhi Arts, Science and Late N. P. Commerce College, Armori, Gadchiroli, MS, India
*Corresponding author Email:

Manuscript Details
Received :28.09.2025
Accepted: 27.10.2025
Published: 30.10.2025

Available online on https://www.irjse.in
ISSN: 2322-0015

Cite this article as:
Rahul A. Parshuramkar and Arpit A. Kharwade. Foundations of Set-Theoretic Granular Computing in Modern Data Analysis , *Int. Res. Journal of Science & Engineering*, 2026, Volume 13(5): 225-233.
https://doi.org/10.5281/zenodo.17166809

Abstract

Modern data analysis faces a fundamental challenge: the exponential growth in data volume has outpaced our ability to extract meaningful patterns using conventional computational approaches. Granular computing, grounded in set-theoretic principles, offers a promising paradigm for addressing this challenge through hierarchical information abstraction. This article presents a comprehensive examination of set-theoretic granular computing, developing a unified framework that treats granules as subsets of a universal set, granular structures as families of such subsets, and relationships between granules through standard set-inclusion. We demonstrate how this framework integrates diverse methodologies including rough sets, formal concept analysis, knowledge spaces, and fuzzy sets. The article explores theoretical foundations through set theory, examines the classification-oriented versus approximation-oriented interpretations of rough sets, and analyzes the integration of fuzzy set theory for handling gradual membership. We propose a synthesis of these approaches within a granular neural network architecture and discuss implications for knowledge discovery, decision-making, and the development of interpretable AI systems. The findings establish granular computing as a bridge between mathematical rigor and practical computational intelligence.

Keywords: Granular Computing, Set Theory, Rough Sets, Formal Concept Analysis, Information Granules, Neural Networks, Fuzzy Sets

1. Introduction

The era of big data has transformed how we approach information processing, yet the sheer volume and complexity of available data present persistent challenges. Traditional computational methods, while powerful, often struggle to manage the cognitive and computational demands of processing millions of data points directly. This has motivated the exploration of alternative paradigms that emphasize abstraction and hierarchical reasoning [1].

Granular computing emerges as a response to this challenge. The fundamental insight is elegant in its simplicity: rather than processing individual data points, we operate on information granules—collections of objects that exhibit similarity in terms of their properties or functional appearance [2]. This approach mirrors human cognitive processes, where we naturally group and classify information at various levels of abstraction.

What makes granular computing particularly compelling is its deep connection to set theory. At its core, a granule is simply a subset of a universal set, and a granular structure is a family of such subsets. The relationships between granules are captured by the standard set-inclusion relation [1,3]. This set-theoretic foundation provides both mathematical rigor and remarkable flexibility, enabling the integration of diverse methodologies that might otherwise appear unrelated.

Several established approaches can be understood as specific instances of this general framework. Rough set theory, formal concept analysis, knowledge spaces, and even aspects of fuzzy set theory all adopt particular models of granular structures [4]. Recognizing these connections allows us to see granular computing not as a new method but as a unifying perspective that reveals the underlying commonalities among these approaches.

This article aims to establish a comprehensive foundation for set-theoretic granular computing and explore its implications for data analysis. We begin by examining the set-theoretic foundations that underlie granular computing, exploring how basic concepts of set membership, inclusion, and operations translate into granular structures. We then develop a unified framework for representing and analyzing granular systems, demonstrating how different conditions on families of subsets yield different types of granular structures [1,3,4].

The discussion turns to the relationship between granular computing and rough set theory, examining both classification-oriented and approximation-oriented interpretations [5]. We explore how fuzzy set theory extends granular computing to handle gradual membership [6] and how granular neural networks integrate these ideas with modern computational architectures [2]. The article concludes with reflections on the implications of this framework for knowledge discovery, decision-making, and the development of interpretable artificial intelligence systems [7][8].

2. Set-Theoretic Foundations

2.1 The Role of Set Theory in Mathematics

Set theory serves as the foundational language of modern mathematics, providing the concepts in terms of which other mathematical ideas are formulated [9]. This role is particularly significant because set theory's primitive notions—set and membership—are simple enough to be intuitive yet powerful enough to support the entire mathematical edifice.

The importance of set theory extends beyond pure mathematics. As Enderton observes, set theory provides an infrastructure that can remain unseen until it is needed, supporting formalization, generalization, and rigorous reasoning across domains [9]. This observation applies equally to data analysis and computational intelligence: set theory provides the conceptual infrastructure that can remain in the background until questions of formalization, generalization, or interpretability arise.

The foundational role of set theory manifests in several ways. First, it provides a universal language for mathematics—a language into which statements from diverse areas can be translated. Second, it establishes a common axiomatic basis that enables proofs and reasoning across domains. The Zermelo-Fraenkel axioms have become the accepted standard for working mathematicians, providing the foundation upon which other mathematical structures are built [9][10].

2.2 Basic Set Operations and Structures

The fundamental operations of set theory provide the building blocks for granular computing. We begin with the concept of the empty set $\emptyset$, having no members at all, and the power set $\mathcal{P}A$, containing all subsets of a given set A. For example, $\mathcal{P}\{0,1\} = \{\emptyset, \{0\}, \{1\}, \{0,1\}\}$. The power set operation is particularly significant because it moves us "up one level in a certain ranking of sets" , enabling the hierarchical structures essential to granular computing [9.10].

The standard set operations—union, intersection, and complement—provide the mechanisms for combining and manipulating granules. The generalized union

$\bigcup\mathcal{A} = \{x \mid x \text{ is a member of some set in } \mathcal{A}\}$
and generalized intersection
$\bigcap\mathcal{A} = \{x \mid x \text{ is a member of every set in } \mathcal{A}\}$

extend these operations to arbitrary families of sets . These generalizations are essential for granular computing [9], where we frequently work with collections of granules.

The extensionality principle—that sets having the same members are identical—underpins the set-theoretic approach to granular computing [9]. This principle ensures that granules are defined by their content rather than their description, providing a clean foundation for reasoning about equivalence and similarity.

2.3 Relations, Functions, and Orders

Relations between sets provide the structure necessary for granular computing. A binary relation $R \subseteq A \times B$ captures relationships between elements of two sets. Equivalence relations—relations that are reflexive, symmetric, and transitive—are particularly important as they induce partitions of sets into equivalence classes, which are natural information granules [10].

The concept of well-ordering extends to the set-theoretic universe, providing the foundation for ranking sets by the number of iterations of the power-set operation needed to construct them . This notion of rank is essential for understanding the hierarchical nature of granular structures.

3. Granular Computing: A Unified Framework

3.1 The Concept of Information Granules

Information granules are the fundamental units of granular computing. A granule can be defined as a subset of a universal set, where the universal set represents the domain of discourse and the subset represents a collection of objects grouped together based on some criterion [1,3].

The concept of information granulation is not merely a technical convenience—it reflects a fundamental cognitive process. As Pedrycz observes, granulation is a "suitable way of abstraction that helps solve problems in a hierarchical fashion as well as convert the original problem in manageable subtasks"[2]. This hierarchical approach mirrors how humans naturally organize information.

Several formal models of information granules have been developed, including sets (interval analysis), rough sets, fuzzy sets, shadowed sets, and probabilistic frameworks [2]. Each model offers different capabilities for representing and manipulating granules, and the choice of model depends on the nature of the data and the requirements of the application.

3.2 Granular Structures

A granular structure is a family of subsets of the universal set [1.3]. The relationships between granules in this structure are given by the standard set-inclusion relation: a granule A is included in granule B if every member of A is also a member of B.

By imposing different conditions on the family of subsets, we can define different types of granular structures . These conditions might include closure properties (such as closure under union or intersection), connectedness properties, or constraints on the relationships between granules. This flexibility allows the framework to accommodate diverse applications.

Several established methodologies can be understood as specific models of granular structures [1,4]:

- **Rough set analysis**: Granules correspond to equivalence classes induced by an indiscernibility relation [5]
- **Formal concept analysis**: Granules correspond to extents of formal concepts [7]
- **Knowledge spaces**: Granules correspond to skill sets or knowledge states
- **Fuzzy set theory**: Granules are fuzzy subsets with graded membership [6]

This recognition of common structure enables the development of general principles and techniques that apply across methodologies.

3.3 Approximations Based on Granular Structures

The notion of approximation is central to granular computing. Given a granular structure and a target concept, we can define approximations that capture what can be known about the concept given the available granularity [5].

Let U be a universal set and let $\mathcal{G}$ be a granular structure on U. For a target concept $X \subseteq U$, we can define various approximations:

- **Lower approximation**: The union of all granules that are entirely contained in X
- **Upper approximation**: The union of all granules that intersect X
- **Boundary region**: The difference between the upper and lower approximations

These approximations capture different aspects of our knowledge about X. The lower approximation represents what can be confidently asserted, the upper approximation represents what is possible, and the boundary region represents uncertainty [1.5].

The granular computing framework provides a common ground for studying these approximations . Different granular structures yield different approximation properties, and the framework enables systematic comparison of these properties.

4. Rough Set Theory and Granular Computing

4.1 Classification-Oriented Rough Sets

Rough set theory provides one of the most developed and influential applications of granular computing. In the classification-oriented interpretation, rough sets are understood primarily as classifiers that partition objects into three regions: positive, negative, and boundary [4,5].

This interpretation focuses on the membership of objects to a concept X under an indiscernibility relation. Objects whose membership is consistent with the available information belong to the positive region, objects whose membership is contradicted by the information belong to the negative region, and objects whose membership is uncertain belong to the boundary region [5].

Formally, for a set $X \subseteq U$ and an equivalence relation R on U, we define:

- **Lower approximation** $\underline{R}X = \{x \in U \mid [x]_R \subseteq X\}$
- **Upper approximation** $\bar{R}X = \{x \in U \mid [x]_R \cap X \neq \emptyset\}$
- **Positive region**: $\underline{R}X$
- **Negative region**: $U \setminus \bar{R}X$
- **Boundary region**: $\bar{R}X \setminus \underline{R}X$

The classification-oriented view is particularly useful for decision-making and rule extraction. It provides a principled way to handle inconsistency and uncertainty, identifying cases where confident classification is possible and cases where additional information is needed.

4.2 Approximation-Oriented Rough Sets

The approximation-oriented interpretation views rough sets differently. Rather than classifying objects, this interpretation focuses on approximating a set X using the elementary sets of a partition [4,5].

In this view, the lower approximation provides the inner approximation of X—the largest union of elementary sets that is contained in X. The upper approximation provides the outer approximation—the smallest union of elementary sets that contains X [5].

The two interpretations—classification-oriented and approximation-oriented—offer complementary perspectives on rough set theory. The classification-oriented view emphasizes decision-making and rule extraction, while the approximation-oriented view emphasizes the structural relationship between concepts and their representations [4,5].

4.3 Rough Sets and Granular Structures

Rough sets can be understood as a specific model of granular structures [1,3]. The elementary sets of the partition—the equivalence classes of the indiscernibility relation—serve as the fundamental granules. The relationships between granules are given by set inclusion, and approximations are defined based on these granules [5].

This perspective reveals connections between rough sets and other granular methodologies. Formal concept analysis, for example, can be understood as a similar framework where concepts are represented as pairs of extents and intents [4,7]. The shared granular structure enables transfer of techniques and insights across methodologies.

Recent developments have extended rough set theory through various generalizations. Variable precision rough sets relax the requirement of exact inclusion, tolerance rough sets replace equivalence relations with tolerance relations, and dominance-based rough sets handle ordered data . These extensions expand the applicability of rough sets to more diverse data types and problem domains [5,6].

5. Fuzzy Sets and Granular Computing

5.1 Fuzzy Sets: Handling Gradual Membership

Classical set theory assumes crisp boundaries between concepts—an element either belongs to a set or it does not. However, many real-world categories have gradual boundaries. Fuzzy set theory addresses this by allowing membership degrees in the interval [0, 1] rather than binary values of 0 or 1 [4,6].
A fuzzy set A in a universe U is characterized by a membership function $\mu_A: U \to [0,1]$,
where $\mu_A(x)$ represents the degree to which element x belongs to A. This graded membership enables more nuanced representation of concepts with fuzzy boundaries.

In granular computing, fuzzy sets provide a framework for granules that have gradual membership. Rather than being either included or not included in a granule, elements can have degrees of inclusion. This is particularly valuable for modeling uncertainty, imprecision, and vagueness in data [4,6].

5.2 Fuzzy Rough Sets: Integrating Two Paradigms

Fuzzy rough set theory integrates the imprecision-handling capabilities of fuzzy sets with the inconsistency-handling capabilities of rough sets [4,6]. This integration creates a powerful framework for managing multiple types of uncertainty in data.

In fuzzy rough sets, the equivalence relations of classical rough sets are replaced by fuzzy similarity relations. The lower and upper approximations become fuzzy sets, with membership degrees indicating the degree of certainty with which an object belongs to a concept [4].

Two main models of fuzzy rough sets have been developed:

1. **Classification-oriented fuzzy rough sets**: Focus on evaluating the consistency of object membership with available information
2. **Approximation-oriented fuzzy rough sets**: Focus on approximating fuzzy sets using elementary fuzzy sets

Both models have been successfully applied to various problems, including feature selection, instance selection, classification, and prediction [6].

5.3 Fuzzy β-Covering Rough Sets

Recent developments have extended fuzzy rough sets through the introduction of fuzzy β-covering models . These models add a confidence-level parameter β that enables flexible granularity regulation [6], addressing limitations of conventional covering models.

The development of fuzzy β-covering rough sets since 2016 has produced eleven model variants organized into neighborhood-driven, logical-operator-enhanced, and structurally-expanded categories [6]. A three-level measurement system has been developed consisting of:

1. Choquet-integral-based fuzzy measures
2. Noise-tolerant discrimination indexes
3. Variable-precision distinguishability indicators

Applications of these models include feature selection, multi-attribute decision-making, outlier detection, and three-way decisions [6]. The parametric nature of β-covering models provides a bridge between fuzzy sets, rough sets, and multigranular analysis.

6. Granular Neural Networks

6.1 From Numeric Processing to Granular Computing

Neural networks have proven remarkably effective for many computational tasks, but they face limitations. Traditional neural networks are "primarily geared toward intensive processing of numeric data" and operate as "black boxes" that are "very difficult to interpret" [2]. As data volumes grow, the computational complexity of direct numeric processing becomes increasingly problematic.

Granular computing offers a way to address these limitations. Granular neural networks process information granules rather than individual data points . This approach provides several advantages:

- **Abstraction**: Granulation provides a suitable way of abstraction that helps solve problems hierarchically
- **Efficiency**: Processing granules rather than individual points reduces computational demands
- **Interpretability**: Granules provide semantic units that can be more readily understood by humans

The design of granular neural networks typically involves two phases: (1) granulation of numeric data, and (2) construction of the neural network based on information granules [2].

6.2 Granulation Frameworks

Several formal frameworks can be used for information granulation in neural networks :

- **Sets (interval analysis)**: Crisp intervals or subsets
- **Fuzzy sets**: Gradual membership functions
- **Rough sets**: Lower and upper approximations
- **Probability**: Probabilistic distributions

The choice of granulation framework depends on the nature of the data and the requirements of the application. Fuzzy sets have been particularly attractive for integration with neural networks because the continuity of membership functions enables gradient-based optimization tools [2.6].

The integration of fuzzy sets with neural networks has produced neurofuzzy systems, which exploit some components of fuzzy set methodology in various ways—at the architectural level, in the design of enhanced learning algorithms, or in interpretation layers [2].

6.3 The Granularity-Learning Trade-off

A key consideration in granular neural networks is the relationship between granularity and learning [2]. The specificity of information granules—how fine or coarse the granulation is—affects both learning complexity and generalization capabilities.

- **Fine granularity**: More specific granules provide more detail but may increase learning complexity and risk overfitting
- **Coarse granularity**: More general granules reduce complexity and may improve generalization but may lose important distinctions [2]

Balancing granularity to optimize performance is a central challenge in designing granular neural networks.

7. Logical and Set-Theoretic Perspectives on Classification

7.1 A Logic-Based View

The classification problem can be approached from a logical perspective that connects naturally to set theory [7]. Given a set of training examples with known class labels, the goal is to predict the class of a new item based on its attribute values.

Let $\mathcal{E}$ be the set of examples for a class C and $\mathcal{E}'$ be the set of counter-examples (examples of other classes). A concept C consistent with the data must satisfy:

$$\mathcal{E} \subseteq C \subseteq \bar{\mathcal{E}'}$$

where $\bar{\mathcal{E}'}$ denotes the complement of the counter-examples [7]. This formulation reveals that the class must include all examples and exclude all counter-examples.

The lower bound $\mathcal{E}$ represents the most specific concept consistent with the data, while the upper bound $\bar{\mathcal{E}'}$ represents the most general concept. The set of all concepts between these bounds constitutes the version space—the set of all hypotheses consistent with the data [7].

7.2 Formal Concept Analysis

Formal concept analysis provides another set-theoretic approach to classification and knowledge representation [4,7]. A formal context is a triple (G, M, I) where G is a set of objects, M is a set of attributes, and $I \subseteq G \times M$ is an incidence relation indicating which objects have which attributes.

A formal concept is a pair (A, B) where $A \subseteq G$, $B \subseteq M$, and:

- Every object in A has all attributes in B
- Every attribute in B is possessed by all objects in A
- A and B are maximal with respect to these conditions

Formal concept analysis provides methods for discovering concepts (clusters of objects with shared attributes) and organizing them into lattices [4,7]. This framework can be seen as a specific model of granular structures, where concepts serve as information granules.

7.3 Rule-Based Classification Using Multi-Soft Set Theory

Soft set theory offers a parameterized approach to set-theoretic reasoning that has been applied to rule-based classification [8,11]. A multi-soft set represents information with multiple parameters, where each parameter defines a subset of objects.

The rule-based classification approach using multi-soft set theory modifies the conventional decision tree induction method, addressing its limitations through the soft computing framework . Key aspects include:

- **Parameterized representation**: Each attribute can be viewed as defining a soft set
- **Rule extraction**: Classification rules are formed by combining subsets through set operations
- **Algorithmic implementation**: The approach has been implemented in Python, demonstrating practical applicability

Applications of this approach span business, agriculture, health, education, and other domains where interpretable rule-based classification is valuable [8,11].

8. Challenges and Future Directions

8.1 Theoretical Challenges

Several theoretical challenges remain for set-theoretic granular computing:

Unified Axiomatic Frameworks: While granular computing provides a common perspective on diverse methodologies, a unified axiomatic framework remains elusive [1,3]. Developing such a framework would facilitate systematic analysis and transfer of techniques.

Computational Complexity: Computing granular structures and approximations can be computationally intensive, particularly for large datasets[5,6]. Efficient algorithms are needed to scale granular methods to modern data volumes.

Interpretability Issues: While granular methods offer improved interpretability compared to black-box approaches, the interpretation of granules and granular structures can itself be challenging [2]. Methods for visualizing and explaining granular results are needed.

Integration with Other Paradigms: The relationship between set-theoretic granular computing and other approaches—such as category theory, probabilistic methods, and deep learning—warrants further investigation [12].

8.2 Emerging Applications

The development of granular computing continues to enable new applications:

Multi-Attribute Decision-Making: Granular methods provide frameworks for handling uncertainty in decision problems with multiple criteria [6].
Outlier Detection: Granular approaches can identify unusual cases that fall in boundary regions or do not fit well into any granule [5,6].
Three-Way Decisions: The three regions of rough sets (positive, negative, boundary) provide a natural framework for decisions with options of acceptance, rejection, and deferral [5,6].

Interpretable AI: Granular computing offers a path toward more interpretable artificial intelligence systems, as granules provide semantic units that can be understood by humans [2].

8.3 Future Research Directions

Several promising directions for future research emerge from this survey:

Dynamic Granular Computing: Developing frameworks that can adapt granular structures as data changes over time.

Hybrid Granulation: Combining multiple granulation frameworks (e.g., fuzzy and rough) to handle complex uncertainty [4,6].

Granular Deep Learning: Exploring the integration of granular computing with deep learning architectures [2.12].

Granular Reasoning: Developing methods for reasoning with and about granules, including operations on granular structures.

Granular Explanation: Using granular structures to explain the decisions of black-box models [2].

9. Conclusion

Set-theoretic granular computing provides a powerful framework for modern data analysis, enabling hierarchical information abstraction that mirrors human cognitive processes. The framework is grounded in the simple concepts of sets, subsets, and set-inclusion, yet it integrates diverse methodologies including rough sets, formal concept analysis, knowledge spaces, and fuzzy sets.

The classification-oriented and approximation-oriented interpretations of rough sets offer complementary perspectives on handling inconsistency and uncertainty. The integration of fuzzy sets extends granular computing to handle gradual membership, while granular neural networks demonstrate the potential of granulation in modern computational architectures.

The logical and set-theoretic perspectives on classification reveal connections among approaches that have been developed independently, suggesting opportunities for synthesis and transfer of insights. Multi-soft set theory provides a parameterized approach to rule-based classification that has demonstrated practical applicability across domains.

As we continue to grapple with the challenges of big data, the demand for efficient, interpretable, and robust methods will only grow. Set-theoretic granular computing offers a principled foundation for meeting this demand, providing both mathematical rigor and practical utility. By recognizing and leveraging the granular structures that underlie our data, we can develop methods that are not only more efficient but also more comprehensible and more trustworthy.

Conflicts of interest: The authors stated that no conflicts of interest.

Correspondence and requests for materials should be addressed to **Arpit A. Kharwade**

Peer review information : IRJSE thanks the anonymous reviewers for their contribution to the peer review of this work. A peer review file is available.

Reprints and permissions information is available at https://www.irjse.in/reprints

References

1. Yao Y, Zhang N, Miao D, Xu F. Set-theoretic approaches to granular computing. Fundamenta Informaticae. 2012;115(2-3):247-264.
2. Pedrycz W. Granular neural networks. Neurocomputing. 2001;36(1-4):205-224.
3. Yao Y, Zhang N, Miao D, Xu F. Set-theoretic approaches to granular computing. zbMATH. 2012;Zbl 1248.68497.
4. Inuiguchi M, Wu WZ, Cornelis C, Verbiest N. Fuzzy-rough Hybridization. Handbook on Fuzzy Sets and Rough Sets.
5. Wang X, Han C, Han D, Fan Q. A Survey of Rough Sets Theory. Journal of Computer-Aided Design & Computer Graphics. 2013.
6. A survey on fuzzy β-covering rough set models, measurement systems, and applications: Advances and prospects from 2015–2025. Fuzzy Sets and Systems. 2025.
7. Dubois D, Prade H. Towards a Logic-Based View of Some Approaches to Classification Tasks. Lecture Notes in Computer Science. 2020.
8. Kottam S, Paul V. Rule-Based Classification using Multi-Soft Set Theory. International Journal of Engineering and Advanced Technology. 2020;9(3):2269-2276.
9. Enderton HB. Set Theory. Encyclopedia of Mathematics. UCLA.
10. van Emden MH. Set-Theoretic Preliminaries for Computer Scientists. Research Report DCS-304-IR. University of Victoria.
11. Kottam S, Paul V. Rule-Based Classification using Multi-Soft Set Theory. 2020.
12. Kainz W, Wu H. Mathematical Foundation of GIS. 2024.

Publisher's Note

IJLSCI remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.